ORDERS OF INFINITY.

Orders of Infinity: the "Infinitärcalcül" of Paul du Bois-Reymond. By G. H. Hardy, F.R.S. Pp. iv+62. (Cambridge: University Press, 1910.) Price 2*s.* 6*d.* net.

THE subject of this tract has been hitherto inaccessible to English readers, and it is not altogether easy to give a brief account of its contents. Perhaps the simplest method is to start from the familiar facts that $\log x$ and e^x both tend to infinity with x, but in very different ways, namely, $(\log x)/x^\delta$ tends to zero, however small δ may be, and e^x/x^Δ tends to infinity, however large Δ may be (it is understood that δ, Δ do not vary with x). These results would be expressed in du Bois-Reymond's notation by the symbols

$$\log x \prec x^\delta \text{ and } e^x \succ x^\Delta;$$

in addition to these two symbols, du Bois-Reymond used another to imply that the ratio of two functions f, ϕ lies between finite limits (when x tends to infinity). But later writers have found it convenient to make the notation rather more precise, and to write

$$f \asymp \phi$$

to imply the relation just mentioned, and further to use $f \sim \phi$ to imply that $\lim (f/\phi) = 1$; there are also other sub-cases for which reference must be made to Mr. Hardy's tract.

The ideas mentioned above lead very naturally to the *logarithmic scale of infinity,* represented by the sequence of functions

$$\ldots, l_3x, l_2x, l_1x, x, e_1x, e_2x, e_3x, \ldots$$

where

$$l_nx = \log(l_{n-1}x), \; l_1x = \log x,$$

and

$$e_nx = \quad (e_{n-1}x), \; e_1x = e^x.$$

This scale has the property that any element tends to infinity more slowly than any positive power δ of the following element, and more rapidly than any positive power Δ of the preceding element; and it is possible to utilise the scale to classify all ordinary functions of analysis. Mr. Hardy has considered (pp. 16–36) the question as to what more or less artificial functions do and do not fall into place in the scale; we must content ourselves with mentioning the comparatively simple function $\Sigma x^\nu/\nu!$ obtained by selecting certain terms from the exponential series; this *does* fit into the scale if ν takes the values $1, 4, 9 \ldots (\nu = n^2)$ but does *not* if ν has the values $1, 8, 27, \ldots (\nu = n^3)$, nor if $\nu = 1, 2, 4, 8, \ldots (\nu = 2^n)$.

In Appendix ii. (pp. 48–57) Mr. Hardy has made a most interesting summary of recent results in analysis, in which du Bois-Reymond's ideas have proved helpful, and readers who are less interested in logic than in results may be advised to turn to this appendix first. Appendix iii. gives a large variety of numerical results to emphasise the amazing rapidity of increase of the logarithmic scale at the upper end, and its corresponding slowness of increase at the lower end.

We may, perhaps, quote the largest number suggested by physical considerations, namely, the number of molecules in the earth, which is found to be of the order

$$1{\cdot}08 \times 10^{51} \text{ or } 42! \text{ or } e_2(4{\cdot}77).$$

This is considerably larger than the number of sodium wave-lengths which light could traverse in geological time, a number of the order $30!$ or $e_2(4{\cdot}32)$. Both of these numbers are quite small when expressed in terms of second order exponentials, and are far smaller than 9^{9^9}, the largest number expressible in terms of three digits; this last number contains 369,693,100 digits when written at full length, and (printed with 16 digits to the inch) would cover more than 350 miles.

T. J. I'A. B.

MAYA ASTRONOMY.

The Numeration, Calendar Systems, and Astronomical Knowledge of the Mayas. By C. P. Bowditch. Pp. xviii+346+xix plates. (Cambridge: University Press, 1910. Privately printed.)

THIS volume offers to the reader "a statement of the knowledge which we possess of the numeration, calendar, and astronomical attainments of [a] wonderful people."

The sources of information are primarily the records of the Mayas themselves; secondarily, the writings of Spaniards and others about the Mayas. The first source may be subdivided into:—

(i) The books of Chilan Balam.
(ii) The codices.
(iii) Inscriptions.

The Book of Chilan Balam of Mani (Mani being the name of a village added to the title for the purposes of identification) was composed before 1595. All books of this class were copies of older manuscripts, with occasional addenda of current interest. They were regarded with superstitious veneration by the village to which they belonged.

The codices, of which three are extant, contain accounts of Mayan histories, ceremonies, sacrifices, and calendar.

Bishop Landa, at Mani, in 1562, carried out so far as possible a "general destruction of everything which related to the ancient life of the nation." He was a Franciscan friar, and subsequently became Bishop of Yucatan. He was a sincere friend and protector to the natives; he has preserved the Maya alphabet, and with it the key to the inscriptions, a service which "wipes out over and over again his faults, which were those of the century."

The Mayas used a series of twenty day-names in an invariable order, Kan, Chicchan, &c., the first following the last without a break, just as the days of the week do with us. The period of twenty days is referred to as a month.

In addition to this, a device was used of counting up to thirteen, and then beginning again, so that the complete cycle becomes 260 days; just as we should have a complete cycle of 210 days if every month contained thirty days, and if it were usual to name only the day of the week and the day of the month without naming the month. The series of 260 days is called Tonalamatl.

As an illustration of the ingenuity that has to be applied in deciphering the results, we transcribe here from p. 28 an example in which we have merely changed the notation for the convenience of printing. Thus we use four symbols, p, q, P, Q, where the original uses lines and dotes either black or red. We have replaced red signs by small letters, black signs by capitals, dots by p or P, lines by q or Q. We use brackets to divide one group of symbols from another. Then we have to decipher (p)(PPQQQ)(q)-(PPPPQQQ)(pqq)(PQ)(pppp)(QQ)(p). The following interpretation may be considered correct, because it makes sense (the process may be compared with solving an equation by trial and error):—Let p or P denote unity, q or Q denote five; then the sentence reads: One, add seventeen, leaves remainder five; add, nineteen, leaves remainder eleven; add six, leaves remainder four; add ten, leaves remainder one. By "leaves remainder" we mean on dividing by thirteen. It is, as we might say, Sunday; in ten days it will be Wednesday; in five more, Monday; in twelve more, Saturday; and in eight more, Sunday again.

With this sample of the contents we must leave the book to our readers. Some will, no doubt, be interested in the problems of decipherment, others in the results obtained; perhaps still more will feel that they cannot be interested in everything, and other problems and other people have greater claims upon their attention. The world at large would regret to see any branch of knowledge die out or remain stationary, and will, in consequence, feel grateful to the author for his labours.

A MONOGRAPH OF DENDROBIUM.

Das Pflanzenreich, Regni vegetabilis conspectus. Edited by A. Engler. N. 50, II., B. 21, Orchidaceæ, Monandræ, Dendrobiinæ. Pars i., genera n. 275–277. By Fr. Kränzlin. Pp. 382. (Leipzig: W. Engelmann, 1910.) Price 19.20 marks.

THE present volume is the forty-fifth of a series of monographs, comprising the "Pflanzenreich," and the third which deals with the great family of orchids. Of the three latter, the few diandrous genera formed the subject of the first, the work of the late Prof. Ernest Pfitzer, while the second volume, begun by Pfitzer, and completed by Dr. Kränzlin, dealt with the small group of the Coelogyninæ. The bulky "Heft" by Dr. Kränzlin, which is the subject of this notice, is devoted to the great genus Dendrobium and its immediate allies. It is evident therefore that there is still very much to be done before we have, what has been a desideratum since the time of Lindley, a complete monograph of this large and important natural order.

The plan of arrangement of tribes and genera adopted in the "Pflanzenreich" is that which was elaborated by Pfitzer in his account of the Orchidaceæ in the "Pflanzenfamilien." Dr. Kränzlin, however, takes a somewhat different view of the limitations of genera. He is here treating of that portion of the section Dendrobiinæ which is characterised by the presence in the anthers of four pollinia without appendages, and in Pfitzer's arrangement included three genera, Latourea (a monotypic genus), Dendrobium (with 300 species), and Aporum (with twelve species). Dr. Kränzlin points out that the first of these was founded on a misconception, and must be regarded as a synonym of the larger genus, in which he also includes the small genus Aporum. On the other hand, he finds reason for resuscitating the very doubtful genus Callista of Loureiro, which depends on a fragmentary specimen of Loureiro's in the British Museum herbarium, and the genera Sarcopodium of Lindley and Desmotrichum of Blume. He also raises to generic rank the sections Inobulbon and Diplocaulobium, and maintains the genus Adrorhizon, founded by Sir Joseph Hooker on a single species from Ceylon.

The number of species admitted is more than double the estimate given by Pfitzer in the "Pflanzenfamilien" in 1889. The great genus Dendrobium includes more than 600 species, which are distributed among ten subgenera, and the grand total of species contained in the seven genera recognised is more than 700. This great increase in number of species is an index of the large and widespread interest which has been taken in the family of orchids during the last twenty years, a period which, by a strange coincidence, starts from the date of the abrupt termination of the work of the younger Reichenbach. During the whole of this period Dr. Kränzlin has been working continuously and steadily on the order, and with the completion of his monograph of one of the largest genera, as well as one of great interest, to botanists and horticulturists, he has earned a new debt of gratitude from workers both in the pure and applied aspects of the science. A. B. R.

ANTHROPOLOGY.

History of Anthropology. By Dr. A. C. Haddon, F.R.S., with the help of A. H. Quiggin. Pp. x+158. (London: Watts and Co., 1910.) Price 1s. net.

THIS is a fascinating little volume, and deals in a masterly manner with the history of anthropology in so far as that can be done within the compass of some 150 pages. Anthropology is now so vast a subject that it is necessary for the individual student, if he wishes to become a specialist, to confine his attention to a comparatively small fraction of the whole, and very often the specialist in one department knows little or nothing of what has been done in other departments. To such specialists this short history will be of the greatest value, and the science of anthropology as a whole will benefit by the coordination of results obtained in different departments.

The authors divide their subject into the two main divisions of physical anthropology and cultural anthropology, and these again are divided into chapters with somewhat eclectic titles, dealing with the more important and interesting sections. We have, for example, chapters on the "Pioneers of Physical Anthropology," "Anthropological Controversies," and "The Unfolding of the Antiquity of Man," under the